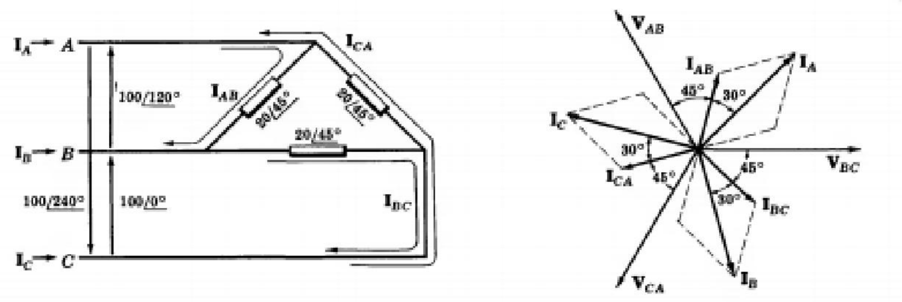
Benha University | Electrical Circuits (2) | Electrical Eng. Dept.
Faculty of Engineering | | 1st year communication
Shoubra | | 10-12 May 2015

Sheet (10) ... Three Phase Systems

1. A three-phase, three-wire 100 volt, ABC system supplies a balanced delta-connected load with impedances of $20\angle 45^\circ$ ohms. Determine the line currents and draw the phasor diagram.

$$I_{AB} = \frac{V_{AB}}{Z} = \frac{100\angle 120^\circ}{20\angle 45^\circ} = 5.0\angle 75^\circ, \quad I_{BC} = \frac{V_{BC}}{Z} = 5.0\angle -45^\circ, \quad I_{CA} = \frac{V_{CA}}{Z} = 5.0\angle 195^\circ$$



To obtain the line currents as shown in the circuit diagram, we apply Kirchhoff's current law at each junction of the load. Thus

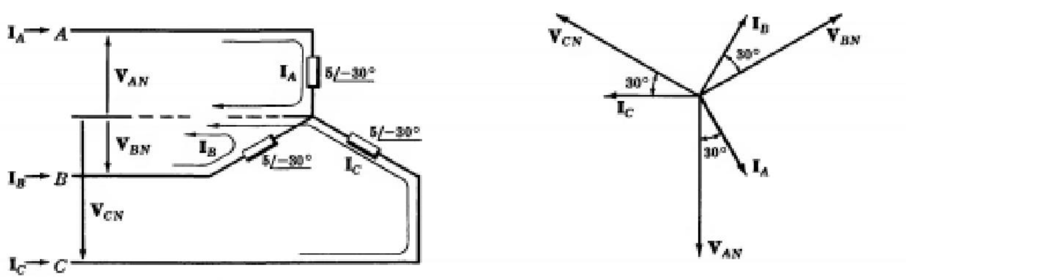
$$I_A = I_{AB} + I_{AC} = 5.0\angle 75^\circ - 5.0\angle 195^\circ = 8.66\angle 45^\circ$$

$$I_B = I_{BA} + I_{BC} = -5.0\angle 75^\circ + 5.0\angle -45^\circ = 8.66\angle -75^\circ$$

$$I_C = I_{CA} + I_{CB} = 5.0\angle 195^\circ - 5.0\angle -45^\circ = 8.66\angle 165^\circ$$

The phasor diagram of phase and line currents is shown in Fig. 14-29 above.

2. Three identical impedances of $5\angle 30^\circ$ ohms are connected in wye to a three-phase, three-wire, 150 volt, CBA system. Find the line currents and draw the phasor diagram.



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With balanced, three-wire, wye-connected systems we may add the neutral conductor as shown in Fig. 14-30. Then the line to neutral voltages, with magnitudes

$$V_{LN} = V_L/\sqrt{3} = 150/\sqrt{3} = 86.6$$

are applied with the phase angles of the *CBA* sequence. The line currents are

$$\mathbf{I}_A = \frac{\mathbf{V}_{AN}}{\mathbf{Z}} = \frac{86.6\angle{-90^\circ}}{5\angle{-30^\circ}} = 17.32\angle{-60^\circ}, \quad \mathbf{I}_B = \frac{\mathbf{V}_{BN}}{\mathbf{Z}} = 17.32\angle{60^\circ}, \quad \mathbf{I}_C = \frac{\mathbf{V}_{CN}}{\mathbf{Z}} = 17.32\angle{180^\circ}$$

The phasor diagram in Fig. 14-31 shows the balanced set of line currents leading the line to neutral voltages by 30°, the angle on the load impedance.

3. Three identical impedances of $10\angle 30^o$ ohms in a wye connection and three identical impedances of $15\angle 0^o$ ohms also in a wye connection are both on the same three-phase, three-wire 250 volt system. Find the total power.

Since both loads are wye-connected, their phase impedances can be put directly on the one-line equivalent circuit as shown in Fig. 14-33. The voltage required in the one-line equivalent circuit is

$$V_{LN} = V_L/\sqrt{3} = 250/\sqrt{3} = 144.5$$

Then the current is

$$\mathbf{I}_L = \frac{144.5\angle{0^\circ}}{10\angle{30^\circ}} + \frac{144.5\angle{0^\circ}}{15\angle{0^\circ}} = 14.45\angle{-30^\circ} + 9.62\angle{0^\circ} = 23.2\angle{-18.1^\circ}$$

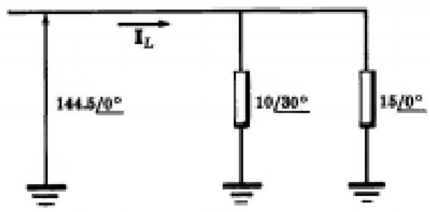

Fig. 14-33

In the power formula $P = \sqrt{3}\, V_L I_L \cos\theta$, θ is the angle of the load impedance when there is a single load. With several loads on the same system, θ is the angle of the equivalent load impedance. In computing the current $\mathbf{I}_L$, both loads were considered and the current was found to lag the voltage by 18.1°. Therefore we know that the equivalent impedance is inductive and has an angle of 18.1°. Then

$$P = \sqrt{3}\, V_L I_L \cos\theta = \sqrt{3}\, 250(23.2) \cos 18.1^\circ = 9530 \text{ w}$$

4. Three identical impedances of $12\angle 30^o$ ohms in a delta connection and three identical impedances of $5\angle 45^o$ ohms in a wye connection are on the same three-phase, three wire, 208 volt, ABC system. Find the line currents and the total power.

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Since the first of the loads is delta-connected we obtain the wye-connected equivalent,

$$Z_Y = Z_\Delta/3 = 12\angle 30^\circ/3 = 4\angle 30^\circ$$

With a line voltage of 208, the line to neutral voltage is $208/\sqrt{3}$ or 120 volts.

The one-line equivalent circuit is shown in Fig. 14-34 with the two load impedances $4\angle 30^\circ$ and $5\angle 45^\circ$ ohms. These impedances can be replaced with an equivalent where

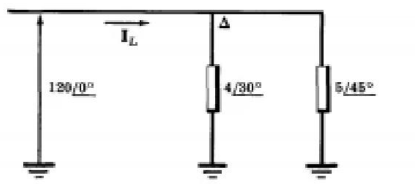

Fig. 14-34

$$Z_{eq} = \frac{4\angle 30^\circ (5\angle 45^\circ)}{4\angle 30^\circ + 5\angle 45^\circ} = 2.24\angle 36.6^\circ$$

Then the current is

$$I_L = \frac{V_{LN}}{Z_{eq}} = \frac{120\angle 0^\circ}{2.24\angle 36.6^\circ} = 53.6\angle -36.6^\circ$$

The voltage V_{AN} in the ABC sequence has a phase angle of 90° and thus $I_A = 53.6\angle(90^\circ - 36.6^\circ) = 53.6\angle 53.4^\circ$. Similarly we find that $I_B = 53.6\angle -66.6^\circ$ and $I_C = 53.6\angle -186.6^\circ$.

The total power

$$P = \sqrt{3}\,V_L I_L \cos\theta = \sqrt{3}\,208(53.6)\cos 36.6^\circ = 15{,}500 \text{ w}$$

5. A three-phase, three-wire, 240 volt, CBA system supplies a delta-connected load in which Zab = $25\angle 90^o$, ZBc = $15\angle 30^o$ and ZCA = $20\angle 0^o$ ohms. Find the line currents and the total power.

Apply the line voltages of the CBA sequence to the delta-connected load in Fig. 14-35, and select the phase currents as shown on the diagram. Then

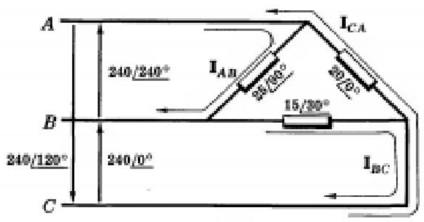

Fig. 14-35

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{240\angle 240^\circ}{25\angle 90^\circ} = 9.6\angle 150^\circ$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{240\angle 0^\circ}{15\angle 30^\circ} = 16.0\angle -30^\circ$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{240\angle 120^\circ}{20\angle 0^\circ} = 12.0\angle 120^\circ$$

Now the line currents are computed in terms of the phase currents.

$$I_A = I_{AB} + I_{AC} = 9.6\angle 150^\circ - 12\angle 120^\circ = 6.06\angle 247.7^\circ$$

$$I_B = I_{BA} + I_{BC} = -9.6\angle 150^\circ + 16\angle -30^\circ = 25.6\angle -30^\circ$$

$$I_C = I_{CA} + I_{CB} = 12\angle 120^\circ - 16\angle -30^\circ = 27.1\angle 137.2^\circ$$

As expected with an unbalanced load, the line currents are not equal.

The power in each phase is calculated as follows.

Impedance $Z_{AB} = 25\angle 90^\circ = 0 + j25$ ohms, $R_{AB} = 0$ and $I_{AB} = 9.6$ amp. Then

$$P_{AB} = I_{AB}^2 R_{AB} = (9.6)^2(0) = 0$$

Impedance $Z_{BC} = 15\angle 30^\circ = 13 + j7.5$ ohms, $R_{BC} = 13$ ohms and $I_{BC} = 16$ amp. Then

$$P_{BC} = I_{BC}^2 R_{BC} = (16)^2(13) = 3330 \text{ w}$$

Impedance $Z_{CA} = 20\angle 0^\circ = 20 + j0$ ohms, $R_{CA} = 20$ ohms and $I_{CA} = 12$ amp. Then

$$P_{CA} = I_{CA}^2 R_{CA} = (12)^2(20) = 2880 \text{ w}$$

The total power is the sum of the power in the phases,

$$P_T = P_{AB} + P_{BC} + P_{CA} = 0 + 3330 + 2880 = 6210 \text{ w}$$

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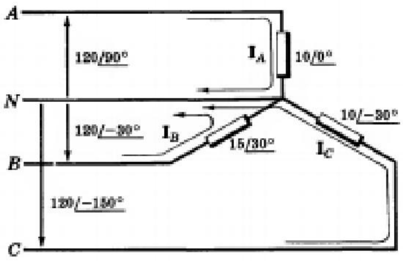
6. A three-phase, four-wire, 208 volt, ABC system supplies a wye-connected load in which ZA= $10\angle 0^o$, ZB= $15\angle 30^o$ and Zc= $10\angle -30^o$ ohms. Find the line currents, the neutral current and the total power.

Apply the line to neutral voltages of the *ABC* sequence to the circuit as shown in Fig. 14-36 and compute the line currents assuming the positive direction toward the load.

$$\mathbf{I}_A = \mathbf{V}_{AN}/\mathbf{Z}_A = (120\angle 90^\circ)/(10\angle 0^\circ) = 12\angle 90^\circ$$

$$\mathbf{I}_B = \mathbf{V}_{BN}/\mathbf{Z}_B = (120\angle -30^\circ)/(15\angle 30^\circ) = 8\angle -60^\circ$$

$$\mathbf{I}_C = \mathbf{V}_{CN}/\mathbf{Z}_C = (120\angle -150^\circ)/(10\angle -30^\circ) = 12\angle -120^\circ$$



The neutral conductor contains the phasor sum of the line currents and if the positive direction is toward the load,

$$\mathbf{I}_N = -(\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C) = -(12\angle 90^\circ + 8\angle -60^\circ + 12\angle -120^\circ) = 5.69\angle 69.4^\circ$$

The impedance $\mathbf{Z}_A = 10 + j0$ ohms passes current $\mathbf{I}_A = 12\angle 90^\circ$ amp, and the power in this phase of the load is $P_A = (12)^2 10 = 1440$ w. Impedance $\mathbf{Z}_B = 15\angle 30^\circ = 13 + j7.5$ contains the current $\mathbf{I}_B = 8\angle -60^\circ$ amp, and the phase power is $P_B = (8)^2 13 = 832$ w. Similarly $\mathbf{Z}_C = 10\angle -30^\circ = 8.66 - j5$ ohms contains $\mathbf{I}_C = 12\angle -120^\circ$ amp and $P_C = (12)^2 8.66 = 1247$ w.

The total power is $P_T = P_A + P_B + P_C = 1440 + 832 + 1247 = 3519$ w.

7. The load impedances of Problem 6 are connected to a three-phase, three-wire, 208 volt, ABC system. Find the line currents and the voltages across the load impedances.

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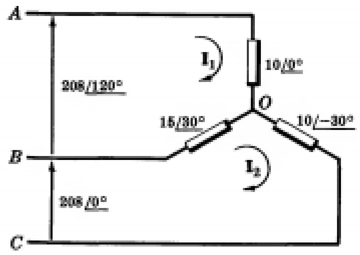

Fig. 14-37

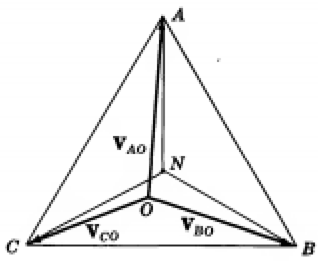

Fig. 14-38

The circuit of Fig. 14-37 shows the two line voltages $\mathbf{V}_{AB}$ and $\mathbf{V}_{BC}$. With the mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$ selected as shown, the matrix form of the mesh current equations is

$$\begin{bmatrix} 10\angle 0^\circ + 15\angle 30^\circ & -15\angle 30^\circ \\ -15\angle 30^\circ & 15\angle 30^\circ + 10\angle -30^\circ \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 208\angle 120^\circ \\ 208\angle 0^\circ \end{bmatrix}$$

from which

$$\mathbf{I}_1 = \frac{5210\angle 90^\circ}{367.5\angle 3.9^\circ} = 14.15\angle 86.1^\circ$$

$$\mathbf{I}_2 = \frac{3730\angle 56.6^\circ}{367.5\angle 3.9^\circ} = 10.15\angle 52.7^\circ$$

The line currents with positive directions toward the load are given in terms of $\mathbf{I}_1$ and $\mathbf{I}_2$ as

$$\mathbf{I}_A = \mathbf{I}_1 = 14.15\angle 86.1^\circ$$

$$\mathbf{I}_B = \mathbf{I}_2 - \mathbf{I}_1 = 10.15\angle 52.7^\circ - 14.15\angle 86.1^\circ = 8.0\angle -49.5^\circ$$

$$\mathbf{I}_C = -\mathbf{I}_2 = 10.15\angle(52.7^\circ - 180^\circ) = 10.15\angle -127.3^\circ$$

Now the voltages across the load impedances are

$$\mathbf{V}_{AO} = \mathbf{I}_A\mathbf{Z}_A = 14.15\angle 86.1^\circ\,(10\angle 0^\circ) = 141.5\angle 86.1^\circ$$

$$\mathbf{V}_{BO} = \mathbf{I}_B\mathbf{Z}_B = 8.0\angle -49.5^\circ\,(15\angle 30^\circ) = 120\angle -19.5^\circ$$

$$\mathbf{V}_{CO} = \mathbf{I}_C\mathbf{Z}_C = 10.15\angle -127.3^\circ\,(10\angle -30^\circ) = 101.5\angle -157.3^\circ$$

A plot of the three voltages $\mathbf{V}_{AO}$, $\mathbf{V}_{BO}$ and $\mathbf{V}_{CO}$ shows the triangle of the ABC sequence when the ends of the phasors are joined by straight lines. Then point N can be added as shown in Fig. 14-38 above.

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